Platform Competition in Two-Sided Markets with Single-Homing

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【Abstract】

Platforms are prevalent in many markets. This paper considers two competing platforms and investigates pricing decisions in two-sided markets with network effects. The purpose of this paper is to examine fundamental concepts such as price-cost margins, market shares, and profitability in equilibrium. A slightly modified Hotelling model suggests the possibility that asymmetric platforms coexist, and a dominant platform with higher market shares earns a higher profit.

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1. Introduction

Following Armstrong (2006), the present paper considers two competing platforms and investigates pricing decisions in two-sided markets with network effects. The purpose of this paper is twofold: firstly, to examine the possibility of asymmetric platforms; and secondly, to attempt to investigate the relationship between market share and profitability. It is shown that a slightly modified Hotelling model suggests the possibility that asymmetric platforms coexist, and a platform with high market shares on both sides earns a higher profit.

Platforms or Intermediaries are prevalent in many markets. Software platforms (Microsoft Windows, Apple iOS, Linux, etc.), electronic payment systems (Visa, MasterCard, etc.), and digital marketplaces (eBay, Amazon, Uber, Airbnb, etc.) are considered as platforms that allow more than two distinct sides or groups of participants to interact and exchange goods or services. For example, Jullien, Pavan and Rysman (2021, p.488) explain that Microsoft manages a three-sided market among hardware providers, software providers, and consumers. One can consider multi-sided markets consisting of more than two sides, but the present paper will focus on two-sided market. My findings provide potential explanations for some observations where asymmetric platforms coexist in two-sided markets.

In two-sided markets, network effects make the participation decision on one side dependent on the participation on the other side. Starting with the seminal papers by Rochet and Tirole (2003) and Armstrong (2006), the literature on two-sided platforms focuses on cross-group network effects. In the presence of cross-groups network effects, platforms are thought to refrain from increasing prices. Suppose that there are two sides, side 1 and side 2. When a consumer on side 1 leaves a platform, network effects are reduced on the other side, inducing lower demand on side 2 and, by a feedback effect, lower value and lower demand on side 1. As a consequence, competition is expected to be more intense. As Jullien, Pavan and Rysman (2021, p.488) point out that ”pricing decisions in the face of indirect network effects are complex because raising the price on one side of the market responds to changes in the participation on the other side. Finding the correct approach to pricing is key to the success of a platform.” In the present paper, I will explicitly solve a unique equilibrium price-cost margins, market shares and profits.

In the literature, some works have discussed the issue of asymmetric equilibria, including Sun and Tse (2007) and Ambrus and Argenziano (2009). Sun and Tse (2007) examine what determines the dominance or coexistence of two-sided platforms using a differential game approach and show that one platform will dominate the market if participants tend to single-home, whereas it is possible for multiple platforms to coexist if participants tend to multi-home. Ambrus and Argenziano (2009, pp.21-23) point out that a wide variety of markets involves two platforms showing a common phenomenon: one platform is larger and cheaper on one side of the market, and one that is larger and cheaper on the other side. They demonstrate that if there is sufficient consumer heterogeneity, then multiple platforms can coexist in equilibrium, and for all asymmetric equilibria, one platform is cheaper and larger on one side and the other is the opposite. Bai and Tang (2022) and Ko and Shen (2021) also study the profitability of platform(s). Gold and Hogendorn (2016) discuss what causes the market to tip in the Armstrong (2006) two sided market model.

The present paper contributes to showing that asymmetric platforms can coexist in a tractable model. To put it simply, the model in the paper assumes that one platform has an advantage only on one side of agents. Unlike Armstrong (2006), the paper assumes that a certain number of agents on that side have already chosen a particular platform. For example, firms may choose a certain platform due to corporate contracts. Jullien and Sand-Zantman (2021, p.3) pick the young for TikTok and cyclists for Strava as examples of platform's differentiation achieved by offering different types of services to attract a specific subset of users. Moreover, that platform can be considered to have an incumbency advantage. Taking these reasons into consideration, the two platforms are not purely horizontally differentiated in the sense that there are captive agents who do not take into account differentiation or transportation cost in the present paper. As far as I know, Rodrigues, Gonçalves and Vasconcelos (2014) have also already used the same approach to examine a competitive impact of pseudo-generics, which is a generic version of a branded product sold by the producer of a branded pharmaceutical in pharmaceuticals markets. Consumers do not incur any transportation costs from the branded drug, and a vertical differentiation exists between the branded drug and generic alternatives. The pseudo-generic produced by the incumbent and a generic produced by an entrant are horizontally differentiated.

The rest of the paper is organized as follows. Section 2 introduces the model with network effects. In Section 3, I consider the case of two competing platforms and examine pricing strategies in the two-sided markets. I derive a unique asymmetric equilibrium. I examine the inter-platform differences on price-cost margins, market shares, and profitsin equilibrium. In Section 4, I make some concluding remarks.

2. The Model

Consider a two-sided market to be one in which at least two distinct sets of agents or sides interact through an intermediary or a *platform* and in which the behavior of each set of agents directly impacts the utility, or the profit of the other set of agents. Platforms facilitating these markets must attract sufficient participation from two different groups in order to maintain the market. The impact of one set of agents on the other, and the resulting feed-back to the first set of agents is referred to as an (indirect) network effect.

As in Section 4 of Armstrong (2006), the current paper considers two competing platforms engaged in differentiated Bertrand competition. There are two groups or two sides of agents, 1 and 2, and there are two platforms, A and B. We consider a two-stage game with observable actions. In the first stage, the two platforms simultaneously set prices (access fees or membership fees) for the two groups of agents. Denote by p_i^k the price of platform k on side i. I assume that platforms do not engage in price discrimination within the sides. In the second stage, the agents in groups 1 and 2 simultaneously choose which platform to join. We assume for some exogeneous reasons that an agent in group 1 and 2 can join at most one platform, that is, agents on both sides are *single-homing*.

Assume that agents on both sides know all prices. Groups 1 and 2 obtain the respective utilities $\{u_1^i, u_2^i\}$ if they join platform i. Agents only care about the number of people joining the platform they choose and the price they have to pay. Denote the volume of participations to platform k on side i by n_i^k. If platform i attracts n_1^i and n_2^i members of the two groups, the utilities on this platform are quasilinear in money and increases in the number of people joining the platform from the other side of the market:

$$u_1^k = g_1^k(n_2^k) - p_1^k; \quad u_2^k = g_2^k(n_1^k) - p_2^k$$

Here, the expression $g_i^k(n_j^k)$ can be interpreted as the strength of network effects on side i. In this paper, I assume that $g_1^k(n_2^k) = \alpha_1 n_2^k$ and $g_2^k(n_1^k) = \alpha_2 n_1^k$. The coefficient α_i is referred to as the agent's *interaction benefit.* Agents are homogeneous so that all agents from the same side have the same interaction benefit α_i on side i.

Platform differentiation can be based on the type of product or service proposed. I use the standard model of competition between two-sided platforms developed by Armstorg (2006) but assume that there are captive agents for platform A on side 2. The product space is assumed to be the unit interval. Platform A is located at the left endpoint of the product space, and platform B at the right endpoint. Agents in a group are assumed to be uniformly located along a unit interval. Platform differentiation on each side i is captured by the parameter t_i, which is usually considered as the transportation cost.

The demand for platform k from side i directly depends not only on the prices (p_i^k, p_i^ℓ) charged on side i, but also on side j's participations. In the paper, the markets are *fully covered*. Then, the full-market-covarage condition on side i is written as

$$D_i^k(p_i^k, p_i^\ell; n_j^k, n_j^\ell) + D_i^\ell(p_i^k, p_i^\ell; n_j^k, n_j^\ell) = 1$$

Each agent on side 1 incurs a cost of joining platform as in the usual Hotelling fashion. As shown in Figure 1, an agent located at θ derives a utility of $u_1^A - t_1\theta$ by joining platform A, and a utility of $u_1^B - t_1(1-\theta)$ by joining platform B. The marginal agent $\hat{\theta}_1$ is indifferent between joining the two platforms. The demands for platforms A and B on side 1 are defined by

$$\begin{aligned} &1 - D_1^B(p_1^A, p_1^B; n_2^A, n_2^B) \\ &\quad = D_1^A(p_1^A, p_1^B; n_2^A, n_2^B) = \mathrm{Prob}(u_1^A - t_1\theta \geqslant u_1^B - t_1(1-\theta)) \end{aligned}$$

Unlike Armstrong (2006), the paper assumes that a certain number of agents in group 2 have already chosen platform A. The two platforms are not purely horizontally differentiated in the sense that there are captive agents of group 2 who prefer platform A without taking into account differentiation or transportation cost.[1] Figure 2 shows that how the marginal agent $\hat{\theta}_2$ is

[1] Rodrigues, Gonvalves and Vasconcelos (2014) also consider such market segmentation in pharmaceuticals markets to examine a competitive impact of pseudo-generic, which is a generic version of a branded product sold by the producer of a branded pharmaceutical.

Figure 1. Market Segmentation on Side 1

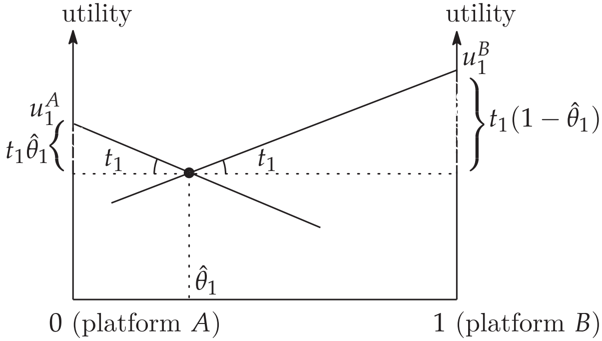
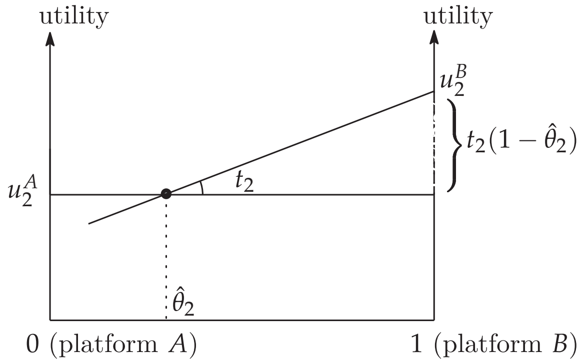


Figure 2. Market Segmentation on Side 2



determined. The demands for platforms A and B on side 2 are defined by

$$\begin{aligned}&1 - D_2^B(p_2^A, p_2^B; n_1^A, n_1^B)\\&\quad = D_2^A(p_2^A, p_2^B; n_1^A, n_1^B) = \text{Prob}(u_2^A \geqslant u_2^B - t_2(1-\theta))\end{aligned}$$

The platforms maximize profits. The revenue of the platform is the sum of the revenues collected from the two groups. Assume that there is no interaction cost between the two sides. Each platform has a per-agent cost f_1 for serving group 1 and f_2 for serving group 2. As will be shown later in

Lemma 3, the numbers of agents on each side are represented as functions of the inter-platform price differences set by the two platforms. Define the sub-profit function on side i of platform k by $\pi_i^k = (p_i^k - f_i)n_i^k$. Platform k's profit is given by

$$\pi^k(p_1^k, p_2^k; p_1^\ell, p_2^\ell) = \pi_1^k + \pi_2^k = (p_1^k - f_1)n_1^k + (p_2^k - f_2)n_2^k$$

Platforms may incur a cost for each interaction between the two sides as discussed in Rochet and Tirole (2003). I ignore the incurred cost from the total transaction volumes.

3. Equilibrium of the Game

Market Shares: The difference between the utilities, $u_i^k - u_i^\ell$, from joining platform i can be written as a function of n_i^k. For each side i, using the fact that $n_j^k + n_j^\ell = 1$ on side $j \neq i$, for each platform k, $\ell \in \{A, B\}$, the utility difference can be written as a function of the price difference as in Eq.(1):

$$\begin{aligned} u_i^k - u_i^\ell &= (\alpha_i n_j^k - p_i^k) - (\alpha_i n_j^\ell - p_i^\ell) \\ &= \alpha_i(n_j^k - (1 - n_j^k)) + (p_i^\ell - p_i^k) \\ &= \alpha_i(2n_j^k - 1) + (p_i^\ell - p_i^k) \end{aligned} \tag{1}$$

Let $(\beta_1^A, \beta_1^B) = (1, 1)$ and $(\beta_2^A, \beta_2^B) = (1, 0)$ and for each group i and platform k, define

$$N_i^k(n_j^k; p_i^k, p_i^\ell) = \beta_i^k + \frac{u_i^k - u_i^\ell}{t_i} = \beta_i^k + \frac{\alpha_i(2n_j^k - 1) + (p_i^\ell - p_i^k)}{t_i} \tag{2}$$

The measure of agents n_1^k and n_2^k from the two sides of the market is formulated as the solution to the two conditions in Lemma 1. That is, n_i^k is determined endogenously through all of the prices.

Lemma 1 (endogenous market segmentation)**.** *The demands on the two sides satisfy the following system of equations:*

$$\begin{cases} n_1^k = \frac{1}{2} N_1^k(n_2^k; p_1^k, p_1^\ell) \\ n_2^k = N_2^k(n_1^k; p_2^k, p_2^\ell) \end{cases}$$

In what follows, suppose that the network externality parameters $\{\alpha_1, \alpha_2\}$ are small compared to the differentiation parameters $\{t_1, t_2\}$ as in Armstrong (2006):

$$2t_1t_2 > (\alpha_1 + \alpha_2)^2$$

Armstrong (2006) employs a similar condition to prevent tipping equilibria.[2] I shall show that the above condition is sufficient for the concavity of the profit functions of the two platforms.

Let $\Delta = t_1t_2 - 2\alpha_1\alpha_2$. The above condition yields that $\Delta > 0$.[3] For each platform k, define

$$\gamma_1^k = \frac{t_1t_2\beta_1^k - 2\alpha_1\alpha_2}{2\Delta} + \frac{\alpha_1t_2(2\beta_2^k - 1)}{2\Delta}; \quad \gamma_2^k = \frac{t_1t_2\beta_2^k - \alpha_1\alpha_2}{\Delta} + \frac{\alpha_2t_1(\beta_1^k - 1)}{\Delta}$$

Lemma 2. *For each side i, $\gamma_i^A + \gamma_i^B = 1$. Furthermore, $\gamma_1^A - \gamma_2^B = \alpha_1t_2/\Delta$ and $\gamma_2^A - \gamma_2^B = t_1t_2/\Delta$.*

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[2] Jullien, Pavan and Rysman (2021, p.510) say that there is *tipping* when all consumers join the same platform. Armstrong (2006) requires that $4t_1t_2 > (\alpha_1 + \alpha_2)^2$ in his model in order to rule out the possibility of tipping. My condition is stronger than his condition because $4t_1t_2 - (\alpha_1 + \alpha_2)^2 > 2(\alpha_1 + \alpha_2)^2 - (\alpha_1 + \alpha_2)^2 = (\alpha_1 + \alpha_2)^2 \geqslant 0$.

[3] Notice that $\Delta = t_1t_2 - 2\alpha_1\alpha_2 > (\alpha_1 + \alpha_2)^2/2 - 2\alpha_1\alpha_2 = (\alpha_1 - \alpha_2)^2/2 \geqslant 0$.

Lemma 3 (market shares)**.** *The number of agents on each side are given as functions of inter-platform price differences:*

$$n_1^k = \gamma_1^k + \frac{2\alpha_1(p_2^\ell - p_2^k) + t_2(p_1^\ell - p_1^k)}{2\Delta}; \quad n_2^k = \gamma_2^k + \frac{\alpha_2(p_1^\ell - p_1^k) + t_1(p_2^\ell - p_2^k)}{\Delta}$$

Equilibrium: According to Lemma 3, the numbers of agents on each side are represented as functions of inter-platform price differences. Platform k's profit is written as

$$\begin{aligned}\pi^k(p_1^k, p_2^k; p_1^\ell, p_2^\ell) &= (p_1^k - f_1)n_1^k + (p_2^k - f_2)n_2^k \\ &= (p_1^k - f_1)\left(\gamma_1^k + \frac{2\alpha_1(p_2^\ell - p_2^k) + t_2(p_1^\ell - p_1^k)}{2\Delta}\right) \\ &\quad +(p_2^k - f_2)\left(\gamma_2^k + \frac{\alpha_2(p_1^\ell - p_1^k) + t_1(p_2^\ell - p_2^k)}{\Delta}\right)\end{aligned}$$

I will show that there is a unique asymmetric equilibrium.[4] The first-order conditions with respect to p_1^k and p_2^k are, respectively,

$$\begin{aligned}0 &= \frac{\partial \pi^k(p_1^k, p_2^k; p_1^\ell, p_2^\ell)}{\partial p_1^k} \\ &= \gamma_1^k + \frac{t_2 f_1 + 2\alpha_2 f_2}{2\Delta} + \frac{2\alpha_1 p_2^\ell + t_2 p_1^\ell}{2\Delta} - \frac{t_2 p_1^k + (\alpha_1 + \alpha_2)p_2^k}{\Delta}\end{aligned} \tag{3}$$

and

$$\begin{aligned}0 &= \frac{\partial \pi^k(p_1^k, p_2^k; p_1^\ell, p_2^\ell)}{\partial p_2^k} \\ &= \gamma_2^k + \frac{t_1 f_2 + \alpha_1 f_1}{\Delta} + \frac{\alpha_2 p_1^\ell + t_1 p_2^\ell}{\Delta} - \frac{(\alpha_1 + \alpha_2)p_1^k + 2t_1 p_2^k}{\Delta}\end{aligned} \tag{4}$$

4　Armstrong (2006) focuses attention on the symmetric equilibrium where each platform offers the same price pair.

The Hessian matrix of the profit function for platform k becomes

$$\mathcal{H} = -\frac{1}{\Delta}\begin{bmatrix} t_2 & \alpha_1+\alpha_2 \\ \alpha_1+\alpha_2 & 2t_1 \end{bmatrix} \tag{5}$$

The Hessian matrix is negative semidefinite if and only if every principal minor of odd order is $\leqslant 0$ and every principal minor of even order is $\geqslant 0$. There are two first-order principal minors and one second-order principal minor. It is obvious that all of the first-order principal minors of $\mathcal{H}$ is strictly negative. If the determinant of the Hessian matrix is non-negative, then the profit function is concave. In fact, the determinant of the Hessian matrix is strictly positive: $\det \mathcal{H} = (2t_1t_2 - (\alpha_1+\alpha_2)^2)/\Delta^2 > 0$. I conclude that the second order principal order is strictly positive. Therefore, the sufficient conditions for concavity of the profit function for platform k are satisfied and thus, the first-order conditions are necessary and sufficient for the optimal solution.

I reformulate the two first-order conditions in Eq.(3) and Eq.(4) in terms of the price-cost margins, $\{p_1^k - f_1, p_2^k - f_2\}$. The reaction functions for each price on each platform constitute a system of four linear equations in four unknowns.

Lemma 4 (best respeonses). *The best responses of platform k given the price-cost margins $\{p_1^\ell - f_1, p_2^\ell - f_2\}$ for platform $\ell \neq k$ are represented as the following matrix form:*

$$\begin{bmatrix} 2t_2 & 2(\alpha_1+\alpha_2) & -t_2 & -2\alpha_1 \\ \alpha_1+\alpha_2 & 2t_1 & -\alpha_2 & -t_1 \end{bmatrix} \begin{bmatrix} p_1^k - f_1 \\ p_2^k - f_2 \\ p_1^\ell - f_1 \\ p_2^\ell - f_2 \end{bmatrix} = \begin{bmatrix} 2\Delta\gamma_1^k \\ \Delta\gamma_2^k \end{bmatrix}$$

Then, the system of equations consisting of the four first-order conditions with four unknowns can be written as the following matrix equation;

$$\begin{bmatrix} 2t_2 & 2(\alpha_1+\alpha_2) & -t_2 & -2\alpha_1 \\ \alpha_1+\alpha_2 & 2t_1 & -\alpha_2 & -t_1 \\ -t_2 & -2\alpha_1 & 2t_2 & 2(\alpha_1+\alpha_2) \\ -\alpha_2 & -t_1 & \alpha_1+\alpha_2 & 2t_1 \end{bmatrix} \begin{bmatrix} p_1^A - f_1 \\ p_2^A - f_2 \\ p_1^B - f_1 \\ p_2^B - f_2 \end{bmatrix} = \begin{bmatrix} 2\Delta\gamma_1^A \\ \Delta\gamma_2^A \\ 2\Delta\gamma_1^B \\ \Delta\gamma_2^B \end{bmatrix},$$

where

$$\begin{cases} 2\Delta\gamma_1^A = t_1t_2 - 2\alpha_1\alpha_2 + \alpha_1 t_2 \\ \Delta\gamma_2^A = t_1t_2 - \alpha_1\alpha_2 \\ 2\Delta\gamma_1^B = t_1t_2 - 2\alpha_1\alpha_2 - \alpha_1 t_2 \\ \Delta\gamma_2^B = -\alpha_1\alpha_2 \end{cases}$$

Denote E by the corresponding coefficient matrix. The determinant of E is given by $\det E = \Delta(9t_1t_2 - 2(2\alpha_1+\alpha_2)(\alpha_1+2\alpha_2))$. The sufficient condition for the concavity of the profit functions of the two platforms, $2t_1t_2 > (\alpha_1 + \alpha_2)^2$, yields that $\det E > 0$.[5]

For example, using Cramer's rule, the price-cost margin for platform A on side 1 is given by

$$p_1^A - f_1 = \frac{1}{\det E} \det \begin{bmatrix} 2\Delta\gamma_1^A & 2(\alpha_1+\alpha_2) & -t_2 & -2\alpha_1 \\ \Delta\gamma_1^B & 2t_1 & -\alpha_2 & -t_1 \\ 2\Delta\gamma_2^A & -2\alpha_1 & 2t_2 & 2(\alpha_1+\alpha_2) \\ \Delta\gamma_2^B & -t_1 & \alpha_1+\alpha_2 & 2t_1 \end{bmatrix}$$

Actually, I can solve for the equilibrium price-cost margins on side 1 as functions of parameters alone. Let $\phi_1(\alpha, t) = 9t_1^2t_2 + 2\alpha_2(2\alpha_1 + \alpha_2)(\alpha_1 +$

5 Notice that $9t_1t_2 - 2(2\alpha_1+\alpha_2)(\alpha_1+2\alpha_2) > 4.5(\alpha_1+\alpha_2)^2 - 2(2\alpha_1+\alpha_2)(\alpha_1+2\alpha_2) = 0.5\alpha_1^2 - \alpha_1\alpha_2 + 0.5\alpha_2^2 = (\alpha_1-\alpha_2)^2/2 \geqslant 0$. Since $\Delta > 0$, it follows that $\det E > 0$.

$2\alpha_2$), which is independent of per-agent transaction costs, f_1 and f_2. Then, the equilibrium price-cost margins in the unique equilibrium is summarized in the following proposition.

Proposition 1 (price-cost margins in equilibrium). *The model with two-sided single-homing has a unique asymmetric equilibrium. Equilibrium price-cost margins on side 1 are given, respectively,*

$$\begin{aligned} p_1^A - f_1 &= \frac{\phi_1(\alpha,t) + t_1(t_2(\alpha_1 - 10\alpha_2) - 2(2\alpha_1+\alpha_2)(\alpha_1+2\alpha_2))}{\det E/\Delta}; \\ p_1^B - f_1 &= \frac{\phi_1(\alpha,t) + t_1(-t_2(\alpha_1 + 8\alpha_2) - 2(2\alpha_1+\alpha_2)(\alpha_1+2\alpha_2))}{\det E/\Delta}, \end{aligned}$$

where $\phi_1(\alpha,\, t) = 9t_1^2t_2 + 2\alpha_2(2\alpha_1 + \alpha_2)(\alpha_1 + 2\alpha_2)$. *Equilibrium price-cost margins on side 2 are given respectively*

$$\begin{aligned} p_2^A - f_2 &= \frac{3t_1t_2(2t_2 - 3\alpha_1) + (\alpha_1+2\alpha_2)(2\alpha_1(2\alpha_1+\alpha_2) - t_2(3\alpha_1+\alpha_2))}{\det E/\Delta}; \\ p_2^B - f_2 &= \frac{3t_1t_2(t_2 - 3\alpha_1) + (\alpha_1+2\alpha_2)(2\alpha_1(2\alpha_1+\alpha_2) - t_2(\alpha_1+\alpha_2))}{\det E/\Delta} \end{aligned}$$

The proof of Proposition 1 is somewhat tedious and thus omitted here. According to Proposition 1, if there is no difference in externality parameters, then the two platforms charge the same price for group 1, and the two platforms will charge different prices on side 2.

Corollary 1 (equilibrium prices). *If there is no difference in externality parameters,* $\alpha_1 = \alpha_2 = \alpha$, *then the two platforms charge the same price for group 1, that is,*

$$p_1^A = f_1 + t_1 - \alpha; \quad p_1^B = f_1 + t_1 - \alpha$$

On the other hand, platform A charges a higher price for group 2:

$$p_2^A = f_2 + \frac{2t_2}{3} - \alpha; \quad p_2^B = f_2 + \frac{t_2}{3} - \alpha$$

Recall that the expressions of market shares are represented as functions of the inter-platform price differences. To end this subsection, I shall derive the expressions of the inter-platform price differentials. The following proposition suggests that the sign of the price differential on side 1 can be either positive or negative, whereas the price differential on side 2 would be positive if network effects are sufficiently weak, that is, the network externality parameters $\{\alpha_1, \alpha_2\}$ are small.

Proposition 2 (inter-platform price differences in equilibrium)**.** *The inter-platform price differences in the unique asymmetric equilibrium are as follows:*

$$\begin{aligned} p_1^A - p_1^B &= \frac{2t_1t_2(\alpha_1 - \alpha_2)}{\det E/\Delta}\,; \\ p_2^A - p_2^B &= \frac{t_2(3t_1t_2 - 2\alpha_1(\alpha_1 + 2\alpha_2))}{\det E/\Delta} \end{aligned}$$

I am ready to solve for the equilibrium market shares using Lemma 1 and Proposition 2. The market share of platform A on side 1 becomes

$$\begin{aligned} n_1^A &= \gamma_1^A + \frac{2\alpha_1(p_2^B - p_2^A) + t_2(p_1^B - p_1^A)}{2\Delta} \\ &= \gamma_1^A + \frac{t_2(2\alpha_1^2(\alpha_1 + 2\alpha_2) - t_1t_2(4\alpha_1 - \alpha_2))}{\det E} \\ &= \frac{9t_1t_2 + (\alpha_1 + 2\alpha_2)(t_2 - 2(2\alpha_1 + \alpha_2))}{2\det E/\Delta} \end{aligned}$$

The market share of platform B on side 1 is as follows:

$$\begin{aligned} n_1^B &= \gamma_1^B + \frac{2\alpha_1(p_2^A - p_2^B) + t_2(p_1^A - p_1^B)}{2\Delta} \\ &= \gamma_1^B + \frac{t_2(t_1t_2(4\alpha_1 - \alpha_2) - 2\alpha_1^2(\alpha_1 + 2\alpha_2))}{\det E} \\ &= \frac{9t_1t_2 - (\alpha_1 + 2\alpha_2)(t_2 + 2(2\alpha_1 + \alpha_2))}{2\det E/\Delta} \end{aligned}$$

On the other hand, the market share of platform A on side 2 becomes

$$\begin{aligned} n_2^A &= \gamma_2^A + \frac{\alpha_2(p_1^B - p_1^A) + t_1(p_2^B - p_2^A)}{\Delta} \\ &= \gamma_2^A + \frac{t_1t_2(2(\alpha_1^2 + \alpha_1\alpha_2 + \alpha_2^2) - 3t_1t_2)}{\det E} \\ &= \frac{6t_1t_2 - (2\alpha_1 + \alpha_2)(\alpha_1 + 2\alpha_2)}{\det E/\Delta} \end{aligned}$$

The market share of platform B on side 1 is as follows:

$$\begin{aligned} n_2^B &= \gamma_2^B + \frac{\alpha_2(p_1^A - p_1^B) + t_1(p_2^A - p_2^B)}{\Delta} \\ &= \gamma_2^B + \frac{t_1t_2(3t_1t_2 - 2(\alpha_1^2 + \alpha_1\alpha_2 + \alpha_2^2))}{\det E} \\ &= \frac{3t_1t_2 - (2\alpha_1 + \alpha_2)(\alpha_1 + 2\alpha_2)}{\det E/\Delta} \end{aligned}$$

It is straightforward to use Lemma 2 to obtain the expressions for the inter-platform market share differences. Since $\gamma_1^A - {\gamma_1}^B = \alpha_1 t_2/\Delta$, it follows that the dominance in terms of market shares between the two platforms on side 1 is equal to

$$n_1^A - n_1^B = \frac{t_2(\alpha_1 + 2\alpha_2)}{\det E/\Delta} > 0$$

Similarly, $\gamma_2^A - \gamma_2^B = t_1t_2/\Delta$. Therefore, the dominance in terms of market shares between the two platforms on side 1 is equal to

$$n_2^A - n_2^B = \frac{3t_1t_2}{\det E/\Delta} > 0$$

As stated in the following proposition, platform A can be considered as a *dominant* platform on both sides.

Proposition 3 (inter-platform market share differences in equilibrium)**.** *The inter-platform market share differences in the unique asymmetric equilibrium are strictly positive on both sides:*

$$n_1^A - n_1^B > 0; \quad n_2^A - n_2^B > 0$$

It remains to examine whether the dominant platform A is more profitable. The expressions for inter-platform profit differences are summarized as the following proposition.

Proposition 4 (inter-platform profit differences in equilibrium)**.** *The inter-platform profit differences in the unique asymmetric equilibrium on the two sides are as follows:*

$$\begin{aligned} \pi_1^A - \pi_1^B &= \frac{t_2(t_1(2\alpha_1 + \alpha_2) - \alpha_2(\alpha_1 + 2\alpha_2))}{\det E/\Delta}; \\ \pi_2^A - \pi_2^B &= \frac{t_2(3t_1(t_2 - \alpha_1) - \alpha_1(\alpha_1 + 2\alpha_2))}{\det E/\Delta} \end{aligned}$$

Moreover, the inter-platform profit difference based on both sides is given by

$$\pi^A - \pi^B = \frac{t_2(t_1(3t_2 - \alpha_1 + \alpha_2) - (\alpha_1 + 2\alpha_2)(\alpha_1 + \alpha_2))}{\det E/\Delta}$$

The signs of the profit differences in Proposition 4 are ambiguous. The following corollaries state that if there is not much of a difference between the strengths of network effects on both sides captured by α_1 and α_2, then it is certain that the dominant platform A is more profitable than platform B.

Corollary 2 (profitability of the dominant platform)**.** *In the unique asym-*

metric equilibrium, if there is no product differentiation, $\alpha_1 = \alpha_2 = \alpha$, *then the inter-platform profit differences are as follows:*

$$\pi_1^A - \pi_1^B = \frac{t_2\alpha(t_1 - \alpha)}{3(t_1t_2 - 2\alpha^2)}; \quad \pi_2^A - \pi_2^B = \frac{t_2(t_1t_2 - t_1\alpha - \alpha^2)}{3(t_1t_2 - 2\alpha^2)}$$

Then, the dominant platform A obtains higher profit than the other platform:

$$\pi^A - \pi^B = \frac{t_2}{3} > 0$$

Corollary 3 (profitability of the dominant platform)**.** *In the unique asymmetric equilibrium, if* $\alpha_1 = \alpha_2 = \alpha$ *and* $t_1 = t_2 = t$, *the inter-platform profit differences are as follows:*

$$\pi_1^A - \pi_1^B = \frac{t\alpha(t - \alpha)}{3(t^2 - 2\alpha^2)}; \quad \pi_2^A - \pi_2^B = \frac{t(t^2 - t\alpha - \alpha^2)}{3(t^2 - 2\alpha^2)}$$

Then, the dominant platform A obtains higher profit than the other platform:

$$\pi^A - \pi^B = \frac{t}{3} > 0$$

4. Conclusions

In this paper, I analyze the possibility of an asymmetric equilibrium in two-sided markets with network effects. I solve a unique asymmetric equilibrium and obtain explicit expressions for price-cost margins, market shares, and profits in the unique asymmetric equilibrium. Platforms can steal each other's agents (customers) by undercutting prices, but this strategy is not necessarily profitable intuitively. I use a sligthly modified Hotelling model in which one platform has an advantage only on one side. It is shown that the platform which has an advantage can be considered as a dominant platform because this platform captures larger market shares on *both* sides. Furthermore, the dominant platform obtains a higher *joint* profit if the difference between externality parameters on both sides is sufficiently small. My findings provide potential explanations for some observations where asymmetric platforms coexist.

The emergence of a dominant platform raises the question that such a dominant platform is good for agents or consumers, so it is natural to investigate its welfare consequences. Moreover, in a recent article, Belleflamme and Peitz (2019, p.1) argue that information about the price charged to the other side is not universally known on many two-sided platforms. They solve the game for perfect Bayesian equilibria with passive beliefs and show that platforms have no incentive to disclose price information in a two-sided single-homing duopoly while a monopoly always has an incentive to inform all participants about prices (Proposition 1 in p.5; Proposition 2 in p.7). It is thus worthwhile to examine platform competition with strategic disclosure. This is a fruitful topic for future research.

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5. Appendix

Proof of Lemma 1:

Firstly, $N_1^A + N_1^B = \beta_1^A + \beta_1^B = 1 + 1 = 2$ and $N_2^A + N_2^B = \beta_2^A + \beta_2^B = 1 + 0 = 1$. For the sake of simplicity, sometimes adopt the shorthand, $N_i^k = N_i^k\ (n_j^k;\ p_i^k, p_i^\ell)$. Find the demands of the side 1. The location of the marginal agent $\hat{\theta}_1$ of group 1 satisfies $u_1^A - t_1\hat{\theta}_1 = u_1^B - t_1(1 - \hat{\theta}_1)$. Solve for $\hat{\theta}_1$ to get

$$\hat{\theta}_1 = \frac{1}{2}\left(1 + \frac{u_1^A - u_1^B}{t_1}\right) = \frac{1}{2}\left(\beta_1^A + \frac{\alpha_1(2n_2^A - 1) + (p_1^B - p_1^A)}{t_1}\right) = \frac{1}{2}N_1^A$$

Recall that agents on side 1 is uniformly distributed over the unit interval. This implies that $D_1^A(p_1^A, p_1^B;\ n_2^A, n_2^B) = \text{Prob}(\theta \leqslant \hat{\theta}_1) = \hat{\theta}_1$. The demand for platform A on side 1 is given $n_1^A = \frac{1}{2}N_1^A$. Furthermore, since $\frac{1}{2}\ (N_1^A + N_1^B) = \frac{1}{2}\ (\beta_1^A + \beta_1^B) = 1$ in Eq.(2), the demand for platform B on side 1 is given by $n_1^B = D_1^B(p_1^A, p_1^B;\ n_2^A, n_2^B) = \text{Prob}(\theta \geqslant \hat{\theta}_1) = 1 - \hat{\theta}_1 = \frac{1}{2}N_1^B$. Therefore, each demand on side 1, n_1^A and n_1^B, can be written as a function the demands on the other side.

Find next the demands of the side 2. The location of the marginal agent $\hat{\theta}_2$ of group 2 satisfies $u_2^A = u_2^B - t_2(1 - \hat{\theta}_2)$. Solve for $\hat{\theta}_2$ to get

$$\hat{\theta}_2 = 1 + \frac{u_2^A - u_2^B}{t_2} = \beta_2^A + \frac{\alpha_2(2n_1^A - 1) + (p_2^B - p_2^A)}{t_2} = N_2^A$$

Therefore, $n_2^A = D_2^A(p_2^A, p_2^B;\ n_1^A, n_1^B) = \text{Prob}(\theta \leqslant \hat{\theta}_2)$, and thus $n_2^A = N_2^A(n_1^A;\ p_2^A, p_2^B)$. Similarly, $n_2^B = D_2^B(p_2^A, p_2^B;\ n_1^A, n_1^B) = \text{Prob}(\theta \geqslant \hat{\theta}_2)$. Since $n_2^A + n_2^B = 1$ and $N_2^A + N_2^B = 1$, it follows that $n_2^B = N_2^B$. Therefore, each demand on side 2, n_2^A and n_2^B, can be written as a function the demands on the other side.

Proof of Lemma 2:

For side 1, $\gamma_1^A + \gamma_1^B = (t_1t_2\ (\beta_1^A + \beta_1^B) - 4\alpha_1\alpha_2 + \alpha_1t_2\ (2\beta_2^A - 1 + 2\beta_2^B - 1))/2\Delta = (2t_1t_2 - 4\alpha_1\alpha_2)/2\Delta = 2\Delta/2\Delta = 1$. For side 2, $\gamma_2^A + \gamma_2^B = (t_1t_2(\beta_2^A +$

$\beta_2^B) - 2\alpha_1\alpha_2 + \alpha_2 t_1\left(\beta_1^A - 1 + \beta_1^B - 1\right))/\Delta = (t_1t_2 - 2\alpha_1\alpha_2)/\Delta = \Delta/\Delta = 1$. In addition, for side 1, $\gamma_1^A - \gamma_1^B = (t_1t_2(\beta_1^A - \beta_1^B) + 2\alpha_1 t_2(\beta_2^A - \beta_2^B))/2\Delta = \alpha_1 t_2/\Delta$. For side 2, $\gamma_2^A - \gamma_2^B = (t_1t_2(\beta_2^A - \beta_2^B) + \alpha_2 t_1(\beta_1^A - \beta_1^B))/\Delta = t_1t_2/\Delta$.

Proof of Lemma 3:

Using Lemma 1, together with the definition of N_i^k, the demands for platform A on both sides consists of the following system of euations:

$$\begin{cases} 2n_1^k = \beta_1^k + \dfrac{\alpha_1(2n_2^k - 1) + (p_1^\ell - p_1^k)}{t_1} \\ n_2^k = \beta_2^k + \dfrac{\alpha_2(2n_1^k - 1) + (p_2^\ell - p_2^k)}{t_2} \end{cases}$$

The above system of equations in terms of n_1^k and n_2^k can be written as the following matrix form with its associated coefficient matrix D:

$$\begin{bmatrix} 2 & -\dfrac{2\alpha_1}{t_1} \\ -\dfrac{2\alpha_2}{t_2} & 1 \end{bmatrix} \begin{bmatrix} n_1^k \\ n_2^k \end{bmatrix} = \begin{bmatrix} \beta_1^k - \dfrac{\alpha_1}{t_1} + \dfrac{p_1^\ell - p_1^k}{t_1} \\ \beta_2^k - \dfrac{\alpha_2}{t_2} + \dfrac{p_2^\ell - p_2^k}{t_2} \end{bmatrix} \tag{6}$$

The determinant of D is strictly positive:

$$\det D = 2 - \left(-\frac{2\alpha_1}{t_1}\right)\left(-\frac{2\alpha_2}{t_2}\right) = \frac{2(t_1t_2 - 2\alpha_1\alpha_2)}{t_1t_2} = \frac{2\Delta}{t_1t_2} > 0$$

Using Cramer's rule, I have the solutions of the system of equations in Eq.(6):

$$n_1^k = \frac{1}{\det D}\det\begin{bmatrix} \beta_1^k - \dfrac{\alpha_1}{t_1} + \dfrac{p_1^\ell - p_1^k}{t_1} & -\dfrac{2\alpha_1}{t_1} \\ \beta_2^k - \dfrac{\alpha_2}{t_2} + \dfrac{p_2^\ell - p_2^k}{t_2} & 1 \end{bmatrix}$$

The numerator of the expression for n_1^k is given by the numerator of the expression for $n_1^k = \det \begin{bmatrix} \beta_1^k - \dfrac{\alpha_1}{t_1} + \dfrac{p_1^\ell - p_1^k}{t_1} & -\dfrac{2\alpha_1}{t_1} \\ \beta_2^k - \dfrac{\alpha_2}{t_2} + \dfrac{p_2^\ell - p_2^k}{t_2} & 1 \end{bmatrix}$

$$= \beta_1^k - \frac{\alpha_1}{t_1} + \frac{p_1^\ell - p_1^k}{t_1} - \left(-\frac{2\alpha_1}{t_1}\right)\left(\beta_2^k - \frac{\alpha_2}{t_2} + \frac{p_2^\ell - p_2^k}{t_2}\right)$$
$$= \beta_1^k - \frac{2\alpha_1\alpha_2}{t_1t_2} + \frac{\alpha_1(2\beta_2^k - 1)}{t_1} + \frac{2\alpha_1(p_2^\ell - p_2^k) + t_2(p_1^\ell - p_1^k)}{t_1t_2}$$

Since $\det D = 2\Delta/t_1t_2$, we have the expression for n_1^k as a function of inter-platform price differences set by the two platforms:

$$n_1^k = \frac{t_1t_2\beta_1^k - 2\alpha_1\alpha_2}{2\Delta} + \frac{\alpha_1t_2(2\beta_2^k - 1)}{2\Delta} + \frac{2\alpha_1(p_2^\ell - p_2^k) + t_2(p_1^\ell - p_1^k)}{2\Delta}$$

Similarly, using Cramer's rule, the numerator of the expression for n_2^k is given by

the numerator of the expression for $n_2^k = \det \begin{bmatrix} 2 & \beta_1^k - \dfrac{\alpha_1}{t_1} + \dfrac{p_1^\ell - p_1^k}{t_1} \\ -\dfrac{2\alpha_2}{t_2} & \beta_2^k - \dfrac{\alpha_2}{t_2} + \dfrac{p_2^\ell - p_2^k}{t_2} \end{bmatrix}$

$$= 2\left(\beta_2^k - \frac{\alpha_2}{t_2} + \frac{p_2^\ell - p_2^k}{t_2}\right) - \left(-\frac{2\alpha_2}{t_2}\right)\left(\beta_1^k - \frac{\alpha_1}{t_1} + \frac{p_1^\ell - p_1^k}{t_1}\right)$$
$$= 2\beta_2^k - \frac{2\alpha_1\alpha_2}{t_1t_2} + \frac{2\alpha_2(\beta_1^k - 1)}{t_2} + \frac{2\alpha_2(p_1^\ell - p_1^k) + 2t_1(p_2^\ell - p_2^k)}{t_1t_2}$$

Dividing the right-hand side by $\det D = 2\Delta/t_1t_2$ to get the expression for n_2^k as a function of inter-platform price differences set by the two platforms:

$$n_2^k = \frac{t_1t_2\beta_2^k - \alpha_1\alpha_2}{\Delta} + \frac{\alpha_2t_1(\beta_1^k - 1)}{\Delta} + \frac{\alpha_2(p_1^\ell - p_1^k) + t_1(p_2^\ell - p_2^k)}{\Delta}$$

Proof of Lemma 4:

For each platform k, $0 = \partial\pi^k(p_1^k, p_2^k)/\partial p_1^k$ implies that

$$
\begin{aligned}
0 &= 2\Delta\gamma_1^k + t_2 f_1 + 2\alpha_2 f_2 + 2\alpha_1 p_2^\ell + t_2 p_1^\ell - 2t_2 p_1^k + (\alpha_1 + 2\alpha_2) p_2^k \\
&= 2\Delta\gamma_1^k - 2t_2(p_1^k - f_1) - 2(\alpha_1 + \alpha_2)(p_2^k - f_2) \\
&\quad + t_2(p_1^\ell - f_1) + 2\alpha_1(p_2^\ell - f_2)
\end{aligned}
$$

Similarly, $0 = \partial\pi^k(p_1^k, p_2^k)/\partial p_2^k$ implies that

$$
\begin{aligned}
0 &= \Delta\gamma_2^k + t_1 f_2 + \alpha_1 f_1 + \alpha_2 p_1^\ell + t_1 p_2^\ell - (\alpha_1 + \alpha_2) p_1^k - 2t_1 p_2^k \\
&= \Delta\gamma_2^k - (\alpha_1 + \alpha_2)(p_1^k - f_1) - 2t_1(p_2^k - f_2) \\
&\quad + \alpha_2(p_1^\ell - f_1) + t_1(p_2^\ell - f_2)
\end{aligned}
$$

Proof of Proposition 2:

It suffices to compute the numerators of the respective expressions for $p_1^A - p_1^B$ and $p_2^A - p_2^B$. By Proposition 1, the inter-platform price difference for group 1 is given by

$$
\begin{aligned}
\text{the numerator of } p_1^A - p_1^B &= t_1(t_2(\alpha_1 - 10\alpha_2)) - t_1(-t_2(\alpha_1 + 8\alpha_2)) \\
&= 2t_1 t_2(\alpha_1 - \alpha_2)
\end{aligned}
$$

Similarly, the inter-platform price difference for group 2 is given by

$$
\begin{aligned}
\text{the numerator of } p_2^A - p_2^B &= 3t_1 t_2(2t_2 - t_2) + t_2(\alpha_1 + 2\alpha_2)(-3\alpha_1 + \alpha_1) \\
&= t_2(3t_1 t_2 - 2\alpha_1(\alpha_1 + 2\alpha_2))
\end{aligned}
$$

【References】

Ambrus A, Argenziano R (2009) Asymmetric networks in two-sided markets. American Economic Journal: Microeconomics 1(1):17–52.

Armstrong M (2006) Competition in two-sided markets. RAND Journal of Economics 37(3):668–691.

Bai J, Tang CS (2022) Can two competing on-demand service platforms be profitable? International Journal of Production Economics 250:1–13.

Belleflamme P, Peitz M (2019) Price disclosure by two-sided platforms. International Journal of Industrial Organization 67:1–19.

Bruno J, Alessandro P, Marc R (2021) Chapter 7 - Two-sided markets, pricing, and network effects. In: Ho K, Hortaçsu A, Lizzeri A (eds) Handbook of Industrial Organization, Volume 4, Elsevier, pp 485–592.

Gold A, Hogendorn C (2016) Tipping in two-sided markets with asymmetric platforms. Economic Analysis and Policy 50:85–90.

Jullien B, Sand-Zantman W (2021) The economics of platforms: A theory guide for competition policy. Information Economics and Policy 54:1–19.

Ko CY, Shen B (2021) Are dominant platforms good for consumers? Economic Inquiry 59(3):1364–1377.

Rochet JC, Tirole J (2003) Platform competition in two-sided markets. Journal of the European Economic Association 1(4):990–1029.

Rodrigues V, Gonçalves R, Vasconcelos H (2014) Anti-competitive impact of pseudo-generics. Journal of Industry, Competition and Trade 14(1):83–98.

Sun M, Tse E (2007) When does the winner take all in two-sided markets? Review of Network Economics 6(1):16–40.